(1) Answer each of the following questions clearly in the space provided.

5 (a) What is the criterion for two elements to be in series?

They have same current flowing.

5 (b) State the Thevenin’s Theorem for DC circuits.

Seen from any two terminals, the circuit is equivalent to a DC voltage source in series with a resistor.

5 (c) Show that an inductor acts as a short circuit in a DC steady state.

\[ V_L = L \frac{dI_n}{dt} = 0 \]

\[ \Rightarrow \text{Short circuit} \]

5 (d) What is meant by a phasor diagram?

Show phasors in the complex plane.
(2) Consider the circuit shown.

(a) How many nodes are in the circuit? Identify them by labeling them.

7 nodes, \(A \rightarrow G\).

(b) Find the Thevenin's equivalent circuit seen by the 25-\(\Omega\) resistor.

\[
30\Omega = 12\Omega \left(\frac{12 + 28}{12 + 28 + 30 + 10}\right) 100
\]

\[
V_{Th} = V_{DC} = \frac{12 + 28 + 30 + 10}{12 + 28 + 30 + 10}
\]

\[
V_{Th} = 50\text{ V}
\]

\[
R_{Th} = (12 + 28) 11 \left(\frac{30 + 10}{30 + 10}\right)
\]

\[
R_{Th} = 20\Omega
\]

(c) Use the above results to find power in the 25-\(\Omega\) resistor.

\[
P = \left(\frac{50}{20 + 25}\right)^2 25 = 30.9\text{ W}
\]
(3) The following circuit was in a DC steady state before the switch closed at \( t = 0 \).

Find the following quantities:

5 (a) \( V_c(0) \).

\[
V_c(0) = \frac{30 - 60}{30 + 60} = +80 \text{ V}
\]

5 (b) \( V_c(\infty) \).

\[
\begin{align*}
60I_1 + 30I_2 &= 60 + 30I_2 = 0 \quad (1) \\
60I_1 - 60 + 30(I_1 - I_2) &= 0 \quad (2)
\end{align*}
\]

\[
I_1 = 0.75 \text{ A} \\
I_2 = 0.25 \text{ A}
\]

\[
V_c(\infty) = 60 + 30I_2 + 30(I_2 - I_1) = 0 \\
V_c(\infty) = 60 - 7.5 + 30(-0.5) \\
V_c(\infty) = 67.5 \text{ V}
\]
(c) The time constant $\tau$ for $t > 0$.

\[ R_{Th} = R_{bc} = \left[ \left( 60 + 11 \right) 30 \right] / 30 \]
\[ R_{Th} = 50 \text{ Ohm} \]
\[ R_{Th} = 18.75 \text{ Ohm} \]
\[ \tau = (18.75 \times 80 \times 10^{-6}) \text{ s} \]
\[ \tau = 1.5 \times 10^{-3} \text{ s} \]

(d) $i_0(t)$ for $t > 0$.

\[ 30 i_0 - v_c(t) + 60 = 0 \]
\[ i_0 = \frac{-60 + v_c(t)}{30} \]
\[ i_0 = -2 + \frac{1}{30} \left[ 67.5 + (80 - 67.5) e^{-t/\tau} \right] \text{ A} \]
\[ i_0 = (0.25 + 0.417 e^{-t/\tau}) \text{ A} \]
(4) Find the following quantities in the circuit shown. The sources operate at 60 Hz.

\[ V_1 = 10 \angle 0^\circ \]
\[ V_2 = -10 \angle 0^\circ \]
\[ \frac{V_1}{10} + \frac{V_2}{-J20} = 0 \]
\[ V_1 \left( \frac{1}{J20} + \frac{1}{10} - \frac{1}{J20} \right) + \frac{V_2}{J20} = -2 \angle 0^\circ \]
\[ 10 \angle 0^\circ \left( \frac{1}{J20} + \frac{1}{10} - \frac{1}{J20} \right) - 10 \angle 0^\circ = -2 \angle 0^\circ \]
\[ I_x \left( 1 + j0.5 \right) = -2 \angle 0^\circ \]
\[ I_x = 1.780 \angle 153.4^\circ \text{ A} \]
\[ I_x = (-1.6 + j0.8) \text{ A} \]

(b) \[ I_0 = -10 \angle 0^\circ \angle 45^\circ + \frac{-10 \angle 0^\circ}{20} \]
\[ I_0 = \tilde{I}_x (-0.5 - j1) - 1 \angle 45^\circ \]
\[ I_0 = 1.02 \angle 28.9^\circ \approx 0.893 + j0.4 \]

(c) The instantaneous power in the dependent source at \( t = 5 \text{ ms} \).
\[ p = (10 \angle 0^\circ) \cdot i_0 = 17.89 \cos (377t + 153.4^\circ) \cdot \]
\[ 1.02 \cos (377t + 28.9^\circ) \]
\[ p(t=5\text{ ms}) = 17.89 (-0.14935) (-0.7303) \]
\[ p = 1.95 \text{ W} \]
(5) In the following circuit, power is transferred to a purely resistive load $R_L$.

(a) Find the matching value of $R_L$ for maximum average power transfer.

$$Z_{Th} = \frac{1}{j100} \left[ 50 - j100 \right]$$
$$= \left(200 + j100\right) \Omega$$
$$= 223.6 \angle 26.57^\circ \ \Omega$$
$$R_L = \left| Z_{Th} \right| = 223.6 \ \Omega$$

(b) Connect the above matching load to A and B and find the equivalent impedance seen by the source. Give your answer in the rectangular form.

$$Z_{eq} = -j100 \left[ 50 + \left( j100 \cdot 223.6 \right) \right]$$
$$= -j100 \left[ 87.27 + 82.33 \right] \ \Omega$$
$$= (110.6 - j78.88) \ \Omega$$
$$= 125.8 \angle -35.51^\circ \ \Omega$$

(c) What is the power factor of the impedance calculated in part (b)?

$$\rho = \cos \left( -35.51^\circ \right) = 0.814 \ \text{(lagging)}$$

(d) With the matching load connected, find the complex power provided by the current source.

$$\tilde{S} = I_{rms}^2 Z_{eq}$$
$$\tilde{S} = (2)^2 \cdot 125.8 \angle -35.51^\circ$$
$$\tilde{S} = 543.2 \angle -35.51^\circ \ \text{VA}$$
$$= (442.2 - j315.5) \ \text{VA}$$