Consider the ideal op amp circuit shown below:

5 (a) How would you categorize this amplifier? Circle the answer below.
   (i) summer  (ii) difference  (iii) Inverting  (iv) Noninverting

10 (b) Find the ratio $v_o/v_x$.

\[ v_x = v_+ = \frac{30}{30+20} v_x = 0.6 v_x \]

\[ 0.6 v_x - v_x + 0.6 v_x - v_o = 0 \]

\[ v_o = 2 (0.6 - 1) v_x + 0.6 v_x \]

\[ \frac{v_o}{v_x} = -0.2 \]

5 (c) Find the resistance seen by the source $v_x$.

\[ I_s = \frac{v_x - 0.6 v_x}{20k} + \frac{v_x - 0.6 v_x}{40k} \]

\[ I_s = 0.03 \times 10^{-3} v_x \]

\[ R_{eq} = \frac{v_x}{I_s} = \frac{100 \Omega}{3} \]
(2) Consider the voltage waveform across an energy-storage element shown below:

(a) If the element is a 200 μF capacitor, find the current waveform in the interval 0 < t < 4 ms.

\[ i = 200 \times 10^{-6} \times \frac{10}{2 \times 10^{-3}} \]
\[ i = 1 \text{ A} \]

\[ 2 < t < 4 \text{ ms}, \quad i = 200 \times 10^{-6} \left( -\frac{10}{2 \times 10^{-3}} \right) \]
\[ i = -1 \text{ A} \]

(b) Find the time when energy stored is 20% of the maximum energy in the capacitor.

\[ 0.2 \ W_c \left( t = 2 \right) = \frac{1}{2} \ C \ V_c^2 \left( t \right) \]
\[ V_c^2 = 0.2 \ (10)^2 = 20 \]
\[ V_c = 4.472 \text{ V} \]
\[ t = 0.894 \text{ s and } 3.106 \text{ s} \]

(c) If the element is a 2-H inductor and there is no energy stored at t = 0, find the current waveform in the interval 0 < t < 2 ms.

\[ i \left( t \right) = 0 + \frac{1}{2} \int_0^t 5 \times 10^{-3} t \ dt \]
\[ = 2.5 \times 10^{-3} \frac{t^2}{2} \]
\[ i \left( t \right) = 1250 \ t^2 \text{ A} \]
\[ i \left( 2 \text{ ms} \right) = 5 \text{ mA} \]

(d) Repeat part (c) for 2 < t < 4 ms.

\[ i \left( t \right) = 5 \times 10^{-3} + \frac{1}{2} \int_2^t \left( 20 - 5 \times 10^{-3} t \right) dt \]

\[ = 5 \times 10^{-3} + \frac{1}{2} \left[ 20t - 2500t^{-2} \right]_2 \times 10^{-3} \]

\[ = 5 \times 10^{-3} + \frac{1}{2} \left[ 20t - 2500t^{-2} \right] - \frac{1}{2} \left[ 20 \times 2 \times 10^{-2} - 2500 \times 9 \times 10^{-6} \right] \]

\[ i \left( t \right) = -10 \times 10^{-3} + 10t - 1250 \ t^2 \text{ (A)} \]
(3) In the following circuit, the switch had been open for a long time until it closed at $t = 0$.

Find the following quantities.

10 (a) $v_c(0)$.

25$i_1$ + 5$i_2$ = +10 -- (1)

$V_o = (i_1 - i_2)$ -- (2)

$V_o = 20i_1$ -- (3)

Solution gives $i_1 = -0.5A$, $i_2 = 4.5A$

$V_c = 9V$

10 (b) $v_c(\infty)$.

$V_o = 10V$

$\frac{V_i}{5} + \frac{V_i}{5} + \frac{V_o}{2} = 0 \Rightarrow 2\frac{V_i}{5} = -\frac{V_o}{2} = -5$

$V_i = -12.5V$

$V_c = \frac{2}{2+3} V_i$

$V_c = -5V$

$\times$
10 (c) The time constant for \( t > 0 \).

\[ R_{Th} = 2 \ln (5 + 3) = 1.6 \Omega \]

\[ T = (1.6) (2) = 3.2 \, s \]

5 (d) \( i_c(t) \) for \( t > 0 \).

\[ v_c(t) = -5 + \left[ 9 + 5 \right] e^{-\frac{t}{3.2}} \, V \]

\[ i_c(t) = 2 \frac{d}{dt} \left[ -5 + 14 e^{-\frac{t}{3.2}} \right] \]

\[ i_c(t) = -8.75 e^{-\frac{t}{3.2}} \, A \]

5 (e) \( i_g(t) \) for \( t > 0 \).

\[ i_c + \frac{v_c}{2} = i_2 \]

\[ i_2 = -8.75 e^{-\frac{t}{3.2}} - \frac{1}{2} \left[ -5 + 14 e^{-\frac{t}{3.2}} \right] \]

\[ i_2 = - \left[ 2.5 + 1.75 e^{-\frac{t}{3.2}} \right] A \]
(4) Answer the following questions in the space provided.

5 (a) Which quantity is continuous in an inductor and why?
\[ V_L = L \frac{d^2 i_L}{dt^2} \]
\[ V_L \text{ is continuous for } i_L \text{ to be finite.} \]

5 (b) How many parameters are used in the dependent source model of an op amp and what are their values in an ideal op amp?
\[ R_{in} = \infty \]
\[ R_{out} = 0 \]
\[ A = \infty \]

5 (c) The voltage and current in an energy storage element are given by
\[ v(t) = 10 \cos 100t \text{ V,} \quad \text{and} \quad i(t) = 2 \sin 100t \text{ A.} \]

Identify the element and find its value.
\[ \frac{di'}{dt} = 200 \cos 100t \]

Must be inductor
\[ V = L \frac{di'}{dt} \implies 10 \cos 100t = L \left(200\right) \cos 100t \]
\[ L = 0.05 \text{ H} \]

5 (d) A 100-\( \mu \text{F} \) capacitor is charged from 10 V to 20 V with a time constant of 10 s. Find the current when the voltage is 15 V.
\[ V_c(t) = 20 + \left[10 - 20\right] e^{-t/10} \]
\[ i_c(t) = +100 \times 10^{-6} \left[-10\right] \left(-\frac{1}{10}\right) e^{-t/10} \]
\[ i_c(t) = 100 \times 10^{-6} e^{-t/10} \text{ A} \]
\[ 15 = 20 + \left(10 - 20\right) e^{-t/10} \]
\[ e^{-t/10} = \frac{20 - 15}{10} = 0.5 \]
\[ i_c(t) = 50 \mu \text{A}. \]