Begin by transforming the Δ-connected resistors \((10 \Omega, 40 \Omega, 50 \Omega)\) to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:

Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

\[
R_1 = \frac{(40)(10)}{10 + 40 + 50} = 4 \Omega; \quad R_2 = \frac{(10)(50)}{10 + 40 + 50} = 5 \Omega; \quad R_3 = \frac{(40)(50)}{10 + 40 + 50} = 20 \Omega
\]
The transformed circuit is shown below:

![Circuit Diagram](image)

The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

\[ R_{eq} = (15 + 5)(1 + 4) + 20 = 20\|5 + 20 = 4 + 20 = 24 \Omega \]

Therefore, the current \( i \) in the 24 V source is given by

\[ i = \frac{24 \text{ V}}{24 \Omega} = 1 \text{ A} \]

Use current division to calculate the currents \( i_1 \) and \( i_2 \). Note that the current \( i_1 \) flows in the branch containing the 15 \( \Omega \) and 5 \( \Omega \) series connected resistors, while the current \( i_2 \) flows in the parallel branch that contains the series connection of the 1 \( \Omega \) and 4 \( \Omega \) resistors:

\[ i_1 = \frac{4}{15 + 5}(i) = \frac{4}{20}(1 \text{ A}) = 0.2 \text{ A}, \quad \text{and} \quad i_2 = 1 \text{ A} - 0.2 \text{ A} = 0.8 \text{ A} \]

Now use KVL and Ohm’s law to calculate \( v_1 \). Note that \( v_1 \) is the sum of the voltage drop across the 4 \( \Omega \) resistor, 4\( i_2 \), and the voltage drop across the 20 \( \Omega \) resistor, 20\( i \):

\[ v_1 = 4i_2 + 20i = 4(0.8 \text{ A}) + 20(1 \text{ A}) = 3.2 + 20 = 23.2 \text{ V} \]

Finally, use KVL and Ohm’s law to calculate \( v_2 \). Note that \( v_2 \) is the sum of the voltage drop across the 5 \( \Omega \) resistor, 5\( i_1 \), and the voltage drop across the 20 \( \Omega \) resistor, 20\( i \):

\[ v_2 = 5i_1 + 20i = 5(0.2 \text{ A}) + 20(1 \text{ A}) = 1 + 20 = 21 \text{ V} \]
P 3.56  [a] Calculate the values of the Y-connected resistors that are equivalent to the 10Ω, 40Ω, and 50Ω Δ-connected resistors:

\[ R_X = \frac{10 \times 50}{10 + 40 + 50} = 5 \Omega; \quad R_Y = \frac{50 \times 40}{10 + 40 + 50} = 20 \Omega; \]

\[ R_Z = \frac{10 \times 40}{10 + 40 + 50} = 4 \Omega \]

Replacing the \( R_2 - R_3 - R_4 \) delta with its equivalent Y gives

Now calculate the equivalent resistance \( R_{ab} \) by making series and parallel combinations of the resistors:

\[ R_{ab} = 13 + 5 + [(8 + 4)\| (20 + 4)] + 7 = 33 \Omega \]

[b] Calculate the values of the \( \Delta \)-connected resistors that are equivalent to the 10Ω, 8Ω, and 40Ω Y-connected resistors:

\[ R_X = \frac{10 \times (8) + (8) \times 40 + (10) \times 40}{8} = \frac{800}{8} = 100 \Omega \]

\[ R_Y = \frac{10 \times 8 + (8) \times 40 + (10) \times 40}{10} = \frac{800}{10} = 80 \Omega \]

\[ R_Z = \frac{10 \times 8 + (8) \times 40 + (10) \times 40}{40} = \frac{800}{40} = 20 \Omega \]
Replacing the $R_2$, $R_4$, $R_5$ wye with its equivalent $\Delta$ gives

Make series and parallel combinations of the resistors to find the equivalent resistance $R_{ab}$:

$100\,\Omega \parallel 50\,\Omega = 33.33\,\Omega$; \quad $80\,\Omega \parallel 4\,\Omega = 3.81\,\Omega$

$\therefore 20\,\Omega \parallel (33.33 + 3.81) = 13\,\Omega$

$\therefore R_{ab} = 13 + 13 + 7 = 33\,\Omega$

[c] Convert the delta connection $R_4 - R_5 - R_6$ to its equivalent wye. Convert the wye connection $R_3 - R_4 - R_6$ to its equivalent delta.
\[
\begin{align*}
25 \parallel 6.25 &= 5 \Omega \\
60 \parallel 30 &= 20 \Omega \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{i}_1 &= \frac{(6)(15)}{(40)} = 2.25 \ A; \quad \mathbf{v}_x = 20 \mathbf{i}_1 = 45 \ \text{V} \\
\mathbf{v}_g &= 25 \mathbf{i}_1 = 56.25 \ \text{V} \\
\mathbf{v}_{6.25} &= \mathbf{v}_g - \mathbf{v}_x = 11.25 \ \text{V} \\
\mathbf{P}_{device} &= \frac{11.25^2}{6.25} + \frac{45^2}{30} + \frac{56.25^2}{15} = 298.6875 \ \text{W}
\end{align*}
\]
P 4.8 \( -6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0 \)

\[ \frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0 \]

Solving, \( v_1 = 120 \text{ V} \); \( v_2 = 96 \text{ V} \)

CHECK:

\[ p_{40\Omega} = \frac{(120)^2}{40} = 360 \text{ W} \]

\[ p_{80\Omega} = \frac{(120 - 96)^2}{8} = 72 \text{ W} \]

\[ p_{120\Omega} = \frac{(96)^2}{120} = 76.8 \text{ W} \]

\[ p_{6A} = -(6)(120) = -720 \text{ W} \]

\[ p_{1A} = (1)(96) = 96 \text{ W} \]

\[ \sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720 \text{ W} \]

\[ \sum p_{\text{dev}} = 720 \text{ W} \text{ (CHECKS)} \]
\[
\begin{align*}
\frac{v_1 - 128}{5} + \frac{v_1}{60} + \frac{v_1 - v_2}{4} &= 0 \\
v_2 - \frac{v_1}{4} + \frac{v_2}{80} + \frac{v_2 - 320}{10} &= 0
\end{align*}
\]

In standard form,
\[
\begin{align*}
v_1 \left( \frac{1}{5} + \frac{1}{60} + \frac{1}{4} \right) + v_2 \left( -\frac{1}{4} \right) &= \frac{128}{5} \\
v_1 \left( -\frac{1}{4} \right) + v_2 \left( \frac{1}{4} + \frac{1}{80} + \frac{1}{10} \right) &= \frac{320}{10}
\end{align*}
\]

Solving, \( v_1 = 162 \) V; \( v_2 = 200 \) V

\[i_a = \frac{128 - 162}{5} = -6.8 \text{ A}\]

\[i_b = \frac{162}{60} = 2.7 \text{ A}\]

\[i_c = \frac{162 - 200}{4} = -9.5 \text{ A}\]

\[i_d = \frac{200}{80} = 2.5 \text{ A}\]

\[i_e = \frac{200 - 320}{10} = -12 \text{ A}\]

\[p_{128V} = -(128)(-6.8) = 870.4 \text{ W (abs)}\]
\[p_{320V} = (320)(-12) = -3840 \text{ W (dev)}\]

Therefore, the total power developed is 3840 W.
P 4.17 [a] \(-25 + \frac{v_1}{40} + \frac{v_1}{160} + \frac{v_1 - v_2}{10} = 0\) so \(21v_1 - 16v_2 + 0i_\Delta = 4000\)

\(\frac{v_2 - v_1}{10} + \frac{v_2}{20} + \frac{v_2 - 84i_\Delta}{8} = 0\) so \(-16v_1 + 44v_2 - 1680i_\Delta = 0\)

\(i_\Delta = \frac{v_1}{160}\) so \(v_1 + (0)v_2 - 160i_\Delta = 0\)

Solving, \(v_1 = 352\) V; \(v_2 = 212\) V; \(i_\Delta = 2.2\) A;

\(i_{\text{depsource}} = \frac{212 - 84(2.2)}{8} = 3.4\) A

\(p_{84i_\Delta} = 84(2.2)(3.4) = 628.32\) W(abs)

\(p_{25A} = -25(352) = -8800\) W(del)

\(\therefore p_{\text{dev}} = 8800\) W

\([b]\) \[\sum p_{\text{abs}} = \frac{(352)^2}{40} + \frac{(352)^2}{160} + \frac{(352 - 212)^2}{10} + \frac{(212)^2}{20} + (3.4)^2(8) + 628.32 = 8800\) W

\(\therefore \sum p_{\text{dev}} = \sum p_{\text{abs}} = 8800\) W

P 4.24
The two node voltage equations are:
\[
\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0 \\
\frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} = 0
\]
Place these equations in standard form:
\[
v_1 \left( \frac{1}{80} + \frac{1}{50} + \frac{1}{40} \right) + v_2 \left( -\frac{1}{40} \right) = \frac{50}{80} \\
v_1 \left( -\frac{1}{40} \right) + v_2 \left( \frac{1}{40} + \frac{1}{200} + \frac{1}{800} \right) = 0.75 + \frac{50}{800}
\]
Solving, \( v_1 = 34 \) V; \( v_2 = 53.2 \) V. Thus, \( v_o = v_2 - 50 = 53.2 - 50 = 3.2 \) V.

POWER CHECK:
\[
i_g = (50 - 34)/80 + (50 - 53.2)/800 = 196 \text{ m A}
\]
\[
p_{50\Omega} = -(50)(0.196) = -9.8 \text{ W}
\]
\[
p_{800\Omega} = (50 - 34)^2/80 = 3.2 \text{ W}
\]
\[
p_{40\Omega} = (53.2 - 34)^2/40 = 9.216 \text{ W}
\]
\[
p_{50\Omega} = 34^2/50 = 23.12 \text{ W}
\]
\[
p_{200\Omega} = 53.2^2/200 = 14.1512 \text{ W}
\]
\[
p_{0.75\Omega} = -(53.2)(0.75) = -39.9 \text{ W}
\]
\[
\sum p_{\text{abs}} = 3.2 + 0.0128 + 9.216 + 23.12 + 14.1512 = 49.7 \text{ W} = \sum p_{\text{del}} = 9.8 + 39.9 = 49.7
\]

P 4.27 Place \( 5v_\Delta \) inside a supernode and use the lower node as a reference. Then
\[
\frac{v_\Delta - 15}{10} + \frac{v_\Delta}{2} + \frac{v_\Delta - 5v_\Delta}{20} + \frac{v_\Delta - 5v_\Delta}{40} = 0
\]
\[
12v_\Delta = 60; \quad v_\Delta = 5 \text{ V}
\]
\[
v_o = v_\Delta - 5v_\Delta = -4(5) = -20 \text{ V}
\]