(1) Answer each of the following questions in the space provided.

3  (a) How does a capacitor behave in a DC steady state and why?

\[ i = C \frac{dV}{dt} = 0 \]

3  (b) An appliance is rated for 120V, 7.5 A, 60 Hz. Can you determine the energy used in kWh in 24 hours?

\[ P = (120 \times 7.5) \times \text{pf} \]

3  (c) Consider a 100-\(\mu\)F capacitor with voltage \(v(t) = 100 \cos(1000\pi t + 30^\circ)\) V. Find the instantaneous current \(i(t)\) at \(t = 0.3\) ms.

\[ i(t) = 100 \times 10^{-6} (\cos(1000\pi t) - 0.3 \times 10^{-3}) A \]

\[ i(0.3\text{ ms}) = -3.12\text{ A} \]

3  (d) Consider the circuits shown below. Which ones are not valid and why?

(a) Invalid, does not satisfy KCL

(b) Valid.
3. (c) State the Principle of Superposition for DC circuits.
In a linear circuit with multiple independent sources, the voltage or current at a location is the algebraic sum of contributions of individual sources considered one at a time.

(2) In the circuit shown, let $I_0 = 3\, \text{A}$.

10. (a) Using only the three laws (Ohm's Law, KCL, KVL), find $I_5$. You are not allowed to write and solve a set of simultaneous equations.

\[ V_{ce} = 20 \times (3) = 60\, \text{V} \Rightarrow I_0 = 2\, \text{A}, \quad I_5 = 5\, \text{A} \]

\[ V_{cd} = 60 + ((5)(6)) = 100\, \text{V} \]

\[ I_3 = \frac{100}{5} = 20\, \text{A} \]

\[ I_2 = 20 + 3 + 2 = 25\, \text{A} \]

\[ V_{ab} = 8I_2 + 5I_3 = 200 + 100 = 300\, \text{V} \]

\[ I_1 = \frac{300}{24} = 12.5\, \text{A} \]

\[ I_8 = 37.5\, \text{A} \]

(b) Find the equivalent resistance seen by the source.

\[ R_{eq} = \frac{V_{ab}}{I_8} = \frac{300}{37.5} = 8\, \text{Ω} \]
Consider the circuit shown below with a variable load resistor.

(a) Find the matching load resistor for maximum power delivery.

\[ I_1 - I_x = 2 \Rightarrow I_1 = I_x + 2 \]
\[ 40I_x + 20I_1 + 24 + 40I_x + 30I_1 = 0 \]
\[ 130I_x = -124 \Rightarrow I_x = -0.9539 \text{ A} \]
\[ I_1 = 1.0462 \text{ A} \]

\[ V_{Th} = V_{ab} = 30I_1 + 40I_x \]
\[ = -6.77 \text{ V} \]

(b) Find the maximum power transferred to the matching load.

\[ I_1 - I_x = 1 \Rightarrow I_1 = (1 + I_x) \]
\[ 20I_x + 40I_1 + 30(1 + I_x) + 40I_x = 0 \]
\[ I_x = -0.2308 \text{ A}, \quad I_1 = 0.7692 \text{ A} \]
\[ V_S = 30I_1 + 40I_x = 13.84 \text{ V} \]

\[ P_{max} = \frac{V^2}{4R_{Th}} = \frac{(-6.77)^2}{4 \times 13.84} = 0.828 \text{ W} \]
(4) Consider the following circuit:

The 100-μF capacitor was initially charged to 2.0 V and immediately connected to the source at \( t = 0 \) via the switch. At \( t = 2 \) s, the switch position was changed to fully discharge the capacitor through the resistor \( R_2 \). The resulting voltage waveform is shown below:

(a) Determine the time constant \( \tau_1 \) in the interval \( 0 < t < 2 \) s.

\[ V(t) = 8 + (2 - 8) e^{t/\tau_1} V \]

\[ G = 8 - 6 e^{-t/\tau_1} \]

\[ \tau_1 = 0.91 \text{ s} \]

\[ V(t) = (8 - 6 e^{-t/0.91}) V \]

(b) Plot the capacitor current \( i(t) \) in the interval \( 0 < t < 2 \) s.

\[ i = 100 \times 10^{-6} \times \frac{6}{0.91} e^{-t/\tau_1} \text{ mA} \]

\[ i = 0.659 e^{-t/0.91} \text{ mA} \]

\[ i(t) \text{ mA} \]

\[ 0 \quad 1 \quad 2 \quad \text{t (s)} \]
(c) Determine the time constant \( \tau_2 \) (in the interval \( t > 2 \text{ s} \)).

\[
V(2) = 8 - 6 e^{-1/t_2} = 7.333 V
\]

\[
V(t) = 7.333 e^{-\frac{t-2}{\tau_2}}
\]

\[
\tau_2 = 1.54 \text{ s}
\]

(d) Find the expression for \( i(t) \) for \( t > 2 \text{ s} \) and plot it in the figure below.

\[
i(t) = 100 \times 10^{-6} \left( -7.333 e^{-\frac{t-2}{1.54}} \right) \, \text{mA}
\]

\[
i(t) = -0.476 e^{-\frac{t-2}{1.54}} \, \text{mA}
\]

(e) Find the energy transferred to the capacitor in the interval \( 0 < t < 3 \text{ s} \).

\[
W(2) = \frac{1}{2} \times 100 \times 10^{-6} \left( 2 \right)^2 = 200 \mu J
\]

\[
U(2) = 7.333 e^{-\frac{1}{\tau_2}}
\]

\[
U(3) = 7.333 e^{-\frac{3}{1.54}} \, V
\]

\[
W(3) = \frac{1}{2} \times 100 \times 10^{-6} \left( 3.831 \right)^2 = 7.34 \mu J
\]

\[
W(3) \approx W(2) = 534 \mu J
\]
(5) Consider the following ideal op amp circuit in an AC steady state:

(a) Show the circuit in frequency domain, showing values of all the phasors and impedances.

\[ V_p = V_n = 0 \]
\[ 10 \text{k} \Omega + j20 \text{k} \Omega = (8+j4) \text{k} \Omega \]
\[ V_o = -\frac{8+j4}{5-j10} \cdot 10 \angle 0^\circ \]
\[ V_o = -j8 \text{ V} = 8 \angle -90^\circ \text{ V} \]
\[ V_o(t) = 8 \cos(10t + 90^\circ) \text{ V} \]

(b) Find \( v_0(t) \).

(c) Find \( i_0(t) \).

\[ i_o = \frac{0 - V_o}{10 \text{k} \Omega} = 0.8 \angle 90^\circ \text{mA} \]
\[ i_o(t) = 0.8 \cos(10t + 90^\circ) \text{ mA} \]
The following circuit has been analyzed and some of the current phasors have been determined as follows:

\[ I_0 = (0.8 - j0.6) \text{ A}, \]
\[ I_1 = (-0.2 - j0.6) \text{ A}, \]
\[ I_2 = (2.4 + j1.2) \text{ A}. \]

Find the following quantities:

(a) The complex power absorbed by the voltage source.

\[ \bar{s}_v = -(\bar{i}_2 + \bar{i}_0) = -(3.2 + j0.6) \text{ A} \]
\[ \bar{s} = \frac{1}{2} 3 \angle 0 \circ \bar{i}_s \]
\[ \bar{s} = \frac{1}{2} (-9.6 + j18) \text{ VA} = (-48 + j9) \text{ VA} \]

Note: Since RMS was not mentioned, full phasors were used.

(b) The reactive power supplied by the current source.

\[ \bar{s} = \frac{1}{2} \bar{v}_s (1 \angle 0 \circ) \ast \bar{i}_1 (1 \angle 0 \circ) \]
\[ \bar{s} = \frac{1}{2} (-4 - j12) \text{ absorbed} \]

\[ Q \text{ (supplied)} = 6 \text{ VAR}. \]