LABORATORY 7

LISSAJOUS PATTERNS

An oscilloscope can be used to observe and measure ac waveforms. It may also be used to observe a voltage-versus-voltage pattern by eliminating the time parameter. The resulting pattern is referred to as a Lissajous pattern. It is the purpose of this experiment to acquaint you with the uses of such patterns and the methods by which they may be observed.

By disconnecting the sweep generator of the oscilloscope and applying two ac waveforms to both the horizontal and vertical amplifiers, you can observe a Lissajous pattern on the screen. If one of the input waveforms is completely known, the frequency, phase, and magnitude relationships between the two waveforms may be measured from the pattern, and the unknown waveform parameters may be calculated as described below.

When both applied waveforms are sinusoidal, the resulting Lissajous pattern may take many forms depending upon the frequency ratio and phase difference between the waveforms. Figure 7.1 shows Lissajous patterns for sinusoids of the same frequency, but varying phase relationships.

![Fig. 7.1](image)

The Lissajous patterns shown in Fig. 7.2 are from sinusoids of varying frequency and phase relationships.

To determine the frequency ratio, draw horizontal and vertical lines through the center of the pattern as shown in Fig. 7.3. The ratio of the number of horizontal axis crossings to the number of vertical axis crossings determines the frequency ratio. This ratio is given as

\[
\frac{f_y}{f_x} = \frac{\text{number of horizontal crossings}}{\text{number of vertical crossings}}
\]  

(1)

where \(f_x\) and \(f_y\) are the frequencies of the two waveforms.
Fig. 7.2 Sine-vs-sine plots (Lissajous figures) for several frequency ratios

For example, in Fig. 7.3 the frequency ratio would be

\[
\frac{f_y}{f_x} = \frac{\text{number of horizontal crossings}}{\text{number of vertical crossings}} = \frac{2}{3}
\]

Thus,

\[
f_y = \frac{2}{3} f_x.
\]
We can obtain the phase angle and magnitude relationships for two sinusoids of the same frequency by using their Lissajous pattern, as shown in Fig. 7.4. The ratio of peak voltages is

\[ \frac{V_x}{V_y} = \frac{M}{A} \quad (2) \]

The phase difference is

\[ \theta = \sin^{-1}\left[\frac{B}{A}\right], \quad (3) \]

and the sinusoids are \( v_x = V_x \sin (\omega t) \) and \( v_y = V_y \sin (\omega t + \theta) \).

(Remember, this is valid only for signals for which \( f_y = f_x \).)

![Fig. 7.4](image)

**Prelab:**

Given two sinusoids of the same frequency,

\[ v_x(t) = C \sin (\omega t) \]

\[ v_y(t) = D \sin (\omega t + \theta) \]

1. Show that,

\[ \frac{v_x^2}{C^2} + \frac{v_y^2}{D^2} - \frac{2v_x v_y \cos(\theta)}{CD} = \sin^2(\theta) \]

**Hint:** Use the trigonometric identities:

\[ \sin (\alpha + \beta) = \sin (\alpha) \cos (\beta) + \sin (\beta) \cos (\alpha) \]

\[ \sin^2 (\alpha) + \cos^2 (\alpha) = 1 \]
2. Sketch $v_v$ vs. $v_x$ for the following cases:

(a) $C = D$, $\theta = 0$

(b) $C = 2D$, $\theta = 0$

(c) $C = 2D$, $\theta = \pi / 2$

**Frequency Relationships Between Signals of Different Frequencies**

Procedure:

1. Connect the 7-V transformer output to the vertical (CH-B oscilloscope input. Connect the function generator output to the horizontal oscilloscope input (CH-A. Make sure the oscilloscope is in X-Y Mode. Note that the transformer output is 7-V ac (Here voltage is measured in RMS (peak/$\sqrt{2}$) and not peak-to-peak.) Also note that the frequency of the transformer output is constant at 60 Hz and cannot be adjusted.

2. Set the function generator to sinusoidal. Adjust the function generator amplitude to get a reasonable pattern on the oscilloscope. Now, adjust the function generator frequency to get a pattern that is for two sinusoids of the same frequency. What is the function generator frequency in hertz? Does it agree with the power-line frequency?

3. By adjusting the frequency of the function generator, find four other frequencies at which the Lissajous pattern is stabilized. By the frequency-ratio method outlined in the text, calculate the function generator frequency. Does your calculated frequency agree with the reading taken from the function generator dial? Explain any differences.

4. Repeat parts 1 and 2 using a triangle wave from the function generator. What is your result? Does the frequency ratio equation from Eq.(1) apply? Is this a “true” Lissajous pattern? Explain.
Magnitude and Phase-Angle Relationships for Two Signals of the Same Frequency

Procedure:

5. Construct the circuit shown in Fig. 7.6. Set the function generator to sinusoidal. Connect the input signal, $V_1$, and the output signal, $V_2$, to CH-B and CH-A of the oscilloscope respectively. Adjust the oscilloscope so that the oscilloscope is displaying two sinusoidal waveforms (the input on CH-B, and the output on CH-A).

From the voltage division, we can find that for this circuit the phase-angle relation is:

$$\theta_2 - \theta_1 = \theta = \tan^{-1}(-\omega RC), \text{ in which } \omega = 2\pi f,$$

and the magnitude relationship is:

$$\frac{|V_2|}{|V_1|} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

6. Using the circuit of Fig. 7.6, measure the phase-angle and magnitude relationships at 20 Hz, 200 Hz, 2 kHz, and 6 kHz. Now calculate the theoretical phase and magnitude and compare the results for each frequency. Are the results in agreement with what you expected?